*Technical Report No. 32-695*

# *Anomalous Diffusion and Instabilities of an Argon Plasma in a Strong Magnetic Field*

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The rate of electron density decay with time in an argon discharge tube energized with a capacitor bank of energy up to 3,000 J is measured by using electron continuum intensity, ion current probe, and microwave techniques. The discharge tube is oriented along a magnetic field up to 30 kG. The electron temperature decay is obtained by the spectral-line intensity ratio method. The oscillation or instability in the plasma is monitored with electrostatic and magnetic probes. The coefficient of ambipolar diffusion is evaluated from the decay data obtained in the diffusion-dominated region. The data thus obtained indicate that the coefficient of ambipolar diffusion agrees with the prediction of the collision theory up to the magnetic field of about 1,000 G. Beyond 1,000 G the onset of oscillations is detected and the coefficient of diffusion is much higher than the collision-theory prediction. For $\mathbf{B} > 1{,}000$ G, the coefficient of diffusion varies as $\mathbf{B}^{-1}$ and the absolute values are quite close to Bohm's. It is thought that the mechanism responsible for the anomalous diffusion is the onset of an instability in the plasma, which at low magnetic field is probably dominated by ion oscillation and at high magnetic field by the so-called universal instability. The critical field for the onset of the instability in the present experiments (pressure of gas $\approx 500\ \mu$ Hg) is about 1,000 G.

Author

## I. INTRODUCTION

The normal-diffusion theory based upon collisions of electrons and ions with neutrals and Coulomb interactions between charged particles predicts a greatly reduced diffusion rate in the direction across a magnetic field. The theory has been confirmed by experiments only under restricted conditions (Ref. 1). It has also been observed that the loss of charged particles from a plasma in a magnetic field is much more rapid than the collision theory predicted (Ref. 2–6).

The rapid transport of charged particles across a magnetic field has been associated with various instabilities in the plasma. The instabilities produce a micro-electric field. It is this field, together with the external applied magnetic field, which causes a particle to drift in the direction perpendicular to the applied magnetic field and along the density gradient.

Several authors have attempted to explain this anomalous effect by making certain assumptions as to the nature of the instabilities. Bohm et al (Ref. 2) proposed the general plasma oscillations; Spitzer (Ref. 7) associated this phenomenon with the ion acoustic waves in the plasma; Drummond and Rosenbluth (Ref. 8) have estimated the coefficient of anomalous diffusion associated with the two-stream instability at ion cyclotron frequencies; Nedospasov and Kadomtsev (Ref. 6) have described the screw convective instability in the positive column; Rudakov and Sagdeev (Ref. 9) and Sagdeev et al (Ref. 10) have formulated the onset of the so-called universal instability in an inhomogeneous plasma; Yoshikawa and Rose (Ref. 11) have established a relation between the coefficient of diffusion and amplitude of density fluctuations in a turbulent plasma. At the present time the available data are not adequate to support or to exclude any of these theories conclusively. In this Report, some results of experimental measurement of plasma-decay phenomena in a short-duration discharge tube in a magnetic field are presented.

## II. EXPERIMENTAL ARRANGEMENT AND MEASURING TECHNIQUES

In these experiments, a column of cylindrical plasma is produced by discharging a capacitor bank through a glass discharge tube filled with argon, situated coaxially in a magnetic coil. After cessation of the discharge current the time rate of decay of electron density is determined by electron continuum radiation, ion current probe, and microwave techniques. The electron temperature decay with time is measured with the spectral-line intensity ratio method. Both electrostatic and magnetic probes are set up to detect disturbances in the plasma. The coefficient of diffusion as a function of magnetic field is deduced from the data and compared with theoretical predictions.

The schematic diagram of the apparatus is shown in Fig. 1. The Pyrex discharge tube, 45 cm long and 3.8 cm in diameter, is pumped down to about $10^{-5}$ mm Hg, purged for at least 24 hr with chemically pure argon gas, and then regulated to the desired pressure. A capacitor bank having the capability to deliver 3,000 J with a conventional three-electrode trigger unit installed is used as an energy source for the discharge. The magnetic coils are energized with a megawatt rectifier bank for a brief duration, but long enough to have a constant magnetic field during the period of the discharge. The light intensities, microwave signals, and probe output are monitored simultaneously with two or more Tektronix oscilloscopes.

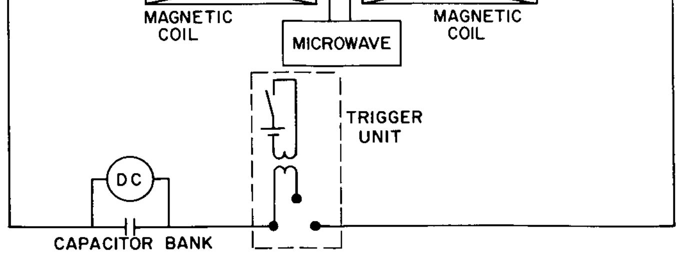


**Fig. 1. Schematic diagram of apparatus arrangement**

### A. Electron Density Measurement

The following three approaches are used to measure the electron density as a function of time.

#### 1. Electron Continuum Intensity

The continuum intensity emitted from a plasma can be expressed as (Ref. 12)

$$\varepsilon = 5.14 \times 10^{-46} Z^2 \frac{N_e^2}{T_e^{1/2}} g \tag{1}$$

where Z is the effective charge of the perturbing ion, $N_e$ the electron number density in $\mathrm{cm}^{-3}$, $T_e$ the electron temperature in °K, $\varepsilon$ the radiant energy in W/cm$^3$ sterad-sec$^{-1}$, and $g$, a Gaunt factor nearly equal to unity, is a very weak function of $T_e$. The accuracy of the constant in Eq. (1) is probably only good within an order of magnitude. However, the functional expression is considered to be correct. In the present experiment the constant, including $g$, is determined from a known value of $N_e$ obtained from microwave measurements, $T_e$ from spectral-line intensity ratio measurements (described in the latter part of this Section and in Section IIB) and independent continuum intensity measurements. The values of $N_e$ are obtained from Eq. (1) by knowing the relative intensity $\varepsilon$ and $T_e$. The electron density thus obtained is the average electron density over the radial distribution. The electron density at the center line of the tube is calculated by assuming that the radial-density variation obeys a Bessel distribution when $\omega_e \tau_e << 1$ or $\nu_{ei} < \nu_{ea}$ (Ref. 13), where $\omega_e$, $\tau_e$, $\nu_{ei}$ and $\nu_{ea}$ are the electron cyclotron frequency, electron collision time, electron-ion collision frequency, and electron-atom collision frequency, respectively. In this case $N_e$ at the center line is equal to 1.22 $N_{e\,meas}$.

Equation (1) is valid only when the following condition is fulfilled. The energy of the free electrons has a Maxwellian distribution, and there are no excited electrons in the ions. This condition is fulfilled in the whole range of present measurements; the details are discussed elsewhere.[1]

The continuum radiation measured in the present experiment is at a wavelength of 5530A where there are no closely spaced spectral lines.

**2. Ion Current Probe**

A conventional Langmuir probe (Ref. 2, 14) is introduced into the tube and is biased negatively to collect all ion current and expel all electrons during the period of measurement. The probe, about 0.3 mm in diameter and 1 mm in length, is cylindrical in shape and is situated at the center line of the tube with its axis oriented in the direction perpendicular to the external applied magnetic field. In such an arrangement the effect of the magnetic field on the ion current measurement is minimized.

**3. Microwave**

Two parallel half-wave flat windows are installed on the wall of the tube. A K-band microwave system is set up to measure the transmission and reflection coefficients of the electromagnetic wave through the plasma (Ref. 15, 16). The cutoff point of the transmitted wave will correspond to the maximum center-line electron density in the discharge tube. This cutoff point is used to calibrate the continuum intensity and ion probe measurements.

### B. Electron Temperature Measurement

The intensities of the atomic argon spectral lines 4259A and 4158A as a function of time are monitored with the use of photomultipliers and an oscilloscope. The spectral sensitivities of the photomultipliers are calibrated with a standardized tungsten-ribbon lamp. The values of $T_e$ as a function of time are evaluated with atomic constants tabulated elsewhere (Ref. 17).

In calculating the electron temperature the radial distribution of the temperature is assumed to be flat owing to the fact that electron-ion recombination takes place near the surface of the wall, thus liberating a substantial amount of energy. This trend is verified in the experimental work of Cooper (Ref. 18).

[1] Chen, Che Jen, *Valid Conditions for the Kramers-Unsöld Continuum Theory in a Non-Equilibrium Plasma,* Technical Report No. 32-707, Jet Propulsion Laboratory, Pasadena, Calif., December 30, 1964.

# III. RESULTS AND DISCUSSION

## A. Electron Temperature

The electron temperature as a function of time is shown in Fig. 2. The portion of the curve below 4,000°K is interpolated by using the equation (Ref. 19)

$$\frac{dT_e}{dt} = -K_0 \frac{2}{3} \frac{m_e}{m_i} \left[\frac{8kT_e}{\pi m_e}\right]^{1/2} (T_e - T_a) N_e \left[ q_{ei} + \frac{N_a}{N_e} q_{ea} \right] \tag{2}$$

where

$q_{ei} = 8\left(\dfrac{e^2}{3kT_e}\right)^2 \ln\left[\dfrac{3}{2e^3} \dfrac{k^3 T_e^3}{\pi N_e}\right]^{1/2}$, the electron-ion collision cross section

$q_{ea} = 10^{-16}$ cm$^2$ for argon, the electron-atom collision cross section

$k$ = Boltzmann constant

$m_e$ = electronic mass

$m_i$ = ionic mass

$e$ = electronic charge

$T_a = T_i$ = atom temperature or ion temperature

The constant $K_0$ is determined from the experimental data in Fig. 2, and is found to be 1.76.

The transfer of the electron energy to the ions and atoms is through elastic electron-ion and electron-atom collisions. It can be shown that the effect of radiation in Eq. (2) is negligible.

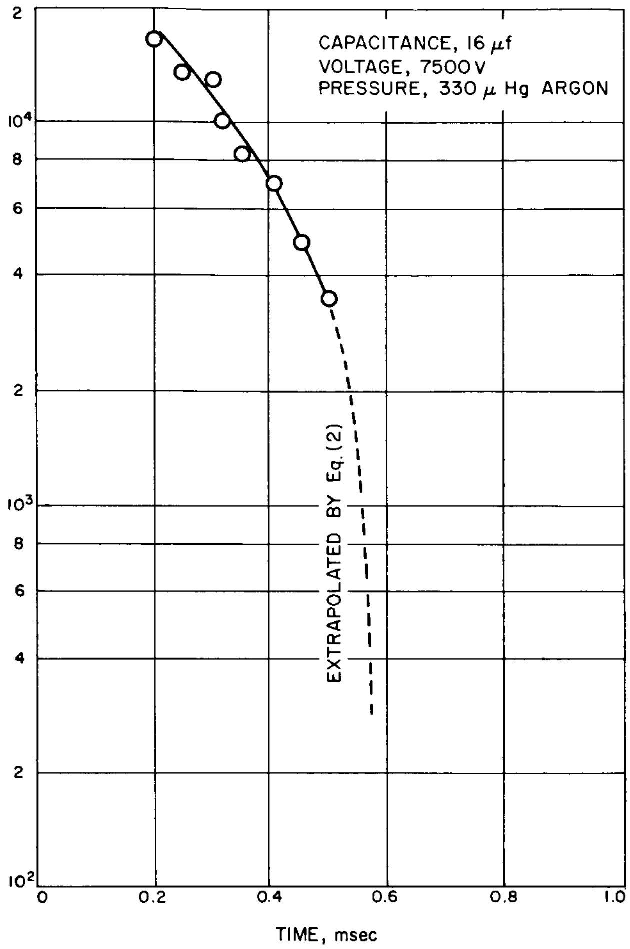


**Fig. 2. Electron temperature as a function of time**

## B. Electron Density

The typical features of the electron decay as a function of time and magnetic field are shown in Fig. 3 and 4. The electron densities measured with continuum radiation and ion current methods agree within 30% if data from both are normalized to the microwave cutoff point.

As can be seen from Eq. (1) the electron density evaluated from continuum measurement is rather insensitive to electron temperature. The error in temperature measurement introduced from the assumption that the radial distribution of electron temperature is flat will be insignificant in the electron density calculations.

The intensity of continuum radiation is too low to be measured within the required accuracy in the present setup when electron density is below $10^{12}$ cm$^{-3}$. The data presented in this Report in the electron density region below $10^{12}$ cm$^{-3}$ are obtained by ion probe measurement.

## C. Ambipolar Diffusion

At the earlier stage of discharge the electron density is high and three-body recombination plays an important role in the loss of charged particles in the plasma. As the plasma decays, the electron density is decreased and the dominating cause for the decay of electron density is shifted from recombination to diffusion until the point is reached where the electron density is low enough that the characteristic mean free path is comparable to the characteristic dimension of the plasma container. At this stage the mechanism responsible for the loss of the charged particles is not collisional diffusion; rather, it is wall recombination.

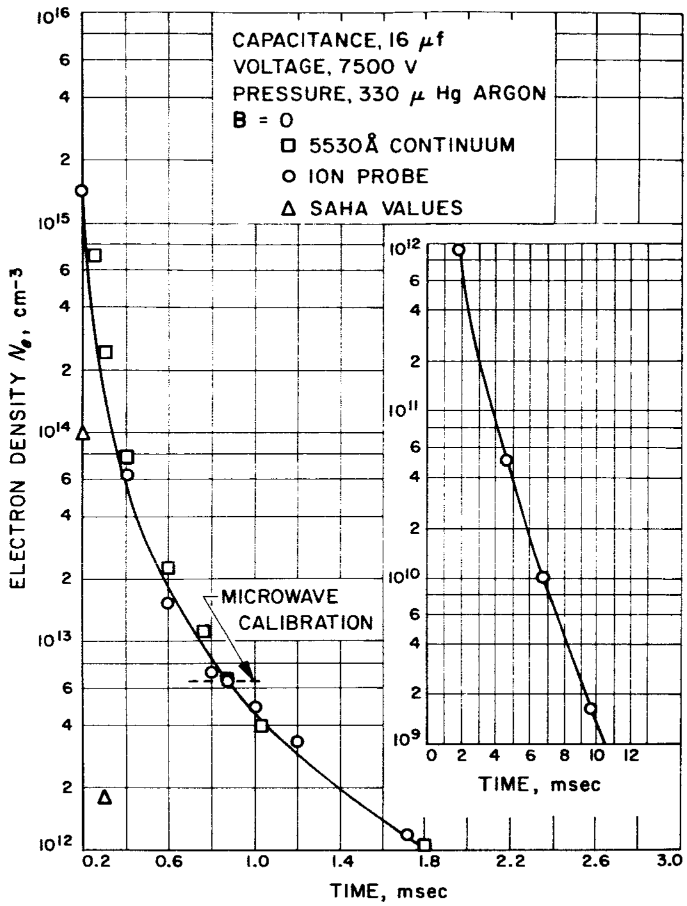


**Fig. 3. Electron density as a function of time**

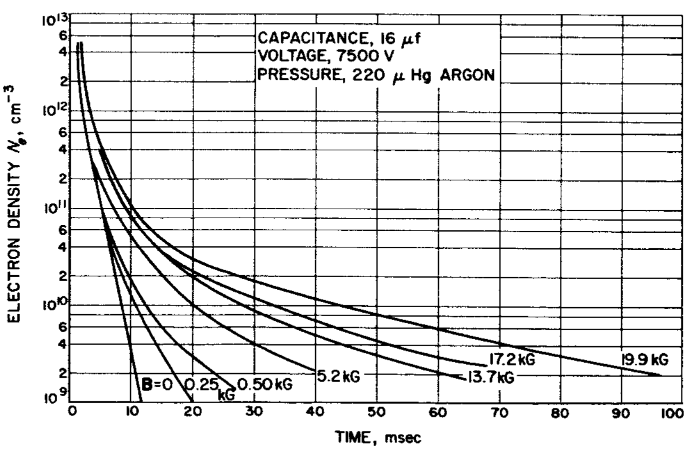


**Fig. 4. Electron density as a function of time and magnetic fields**

Experimental data for three-body recombination of argon covering ranges of electron density and temperature comparable to that in the present experiment do not exist in published literature. Byron et al (Ref. 20), and Bates et al (Ref. 21), have presented the theory of three-body recombination for hydrogenlike gases for a wide range of $N_e$ and $T_e$. By using the results of Ref. 20 and considering the actual physical situation in the present experiment, Fig. 5 is constructed to indicate the range of electron density within which the collisional diffusion is the dominating mechanism responsible for the decay of electron density in the discharge tube. In Fig. 5, $\tau_r$ is the $e$-fold characteristic time for three-body recombination, $\tau_D$ is that for diffusion. Within the shaded area, $1/\tau_r$ is kept below 1% of $1/\tau_D$ and the electron mean free path is less than 1/10 of the radius of the discharge tube. The range of the electron density for the collision-diffusion-dominated region at $T_e = 300°K$ and initial tube pressure of 220 $\mu$ Hg is about from $4 \times 10^9$ to $2 \times 10^{10}$ cm$^{-3}$. The diffusion data presented in this Report are evaluated from the measurement in such a region.

Golant (Ref. 22) has calculated the directional motion of charged particles across a magnetic field in a three-component plasma by using conventional kinetic equations. In his calculation both charge-neutral collisions and Coulomb interactions between charged particles are taken into consideration. Under the assumptions that the effect of ion-electron collisions on the motion of the

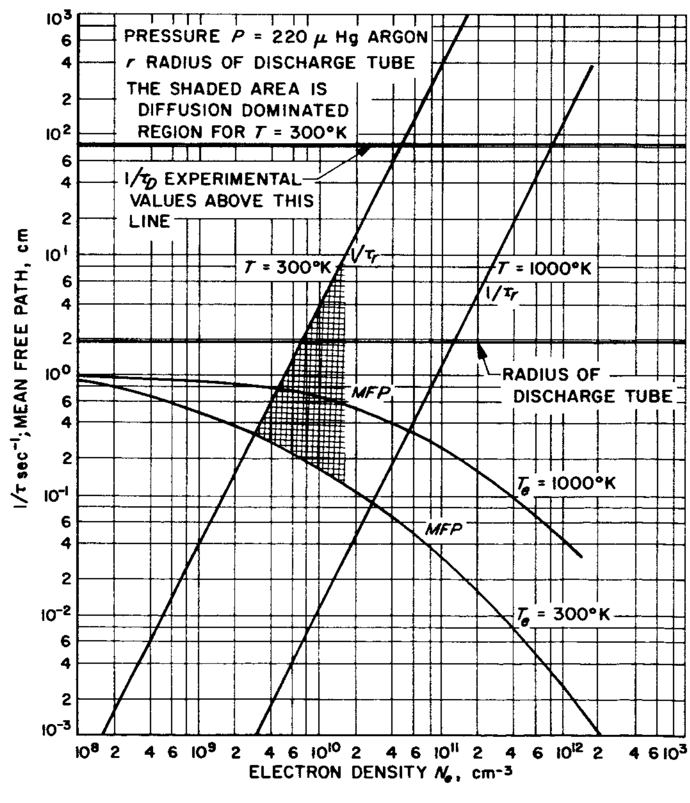


**Fig. 5. Collision-diffusion-dominated region**

ion may be neglected, or $m_e \nu_{ei} << m_i \nu_{ia}$ holds, and $\omega_e >> \nu_{ei} + \nu_{ea}$, his results for the coefficient of ambipolar diffusion of charged particles across a magnetic field can be written as

$$D_a^H = \frac{D_a^0}{1 + \dfrac{\omega_i \omega_e}{(\nu_{ei} + \nu_{ea})\,\nu_{ia}}} \tag{3}$$

where $D_a^0$ is the coefficient of diffusion without magnetic field and has the form

$$D_a^0 = \frac{k\,(T_i + T_e)}{m_i \nu_{ia}}$$

where

$\nu_{ei}$ = electron-ion collision frequency

$\nu_{ea}$ = electron-atom collision frequency

$\nu_{ia}$ = ion-atom collision frequency

and the other symbols have the same meaning as before.

The experimental values of $D_a^0$ and $D_a^H$ are obtained by calculating $(1/N_e)(dN_e/dt)$ from experimental data and using the expression

$$D_a^H = \frac{1}{N_e}\frac{dN_e}{dt}\Lambda^2 = \frac{\Lambda^2}{\tau_D} \tag{4}$$

where $\Lambda$ is the diffusion length and is equal to (radius of tube)/2.40 for the Bessel distribution of electron density in the tube.

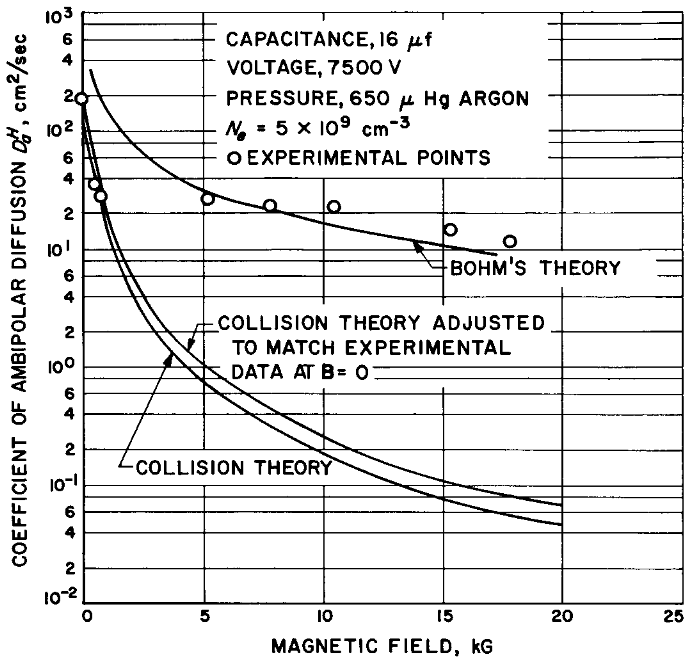


**Fig. 7. Coefficient of ambipolar diffusion as a function of magnetic field at pressure = 650 μ Hg argon**

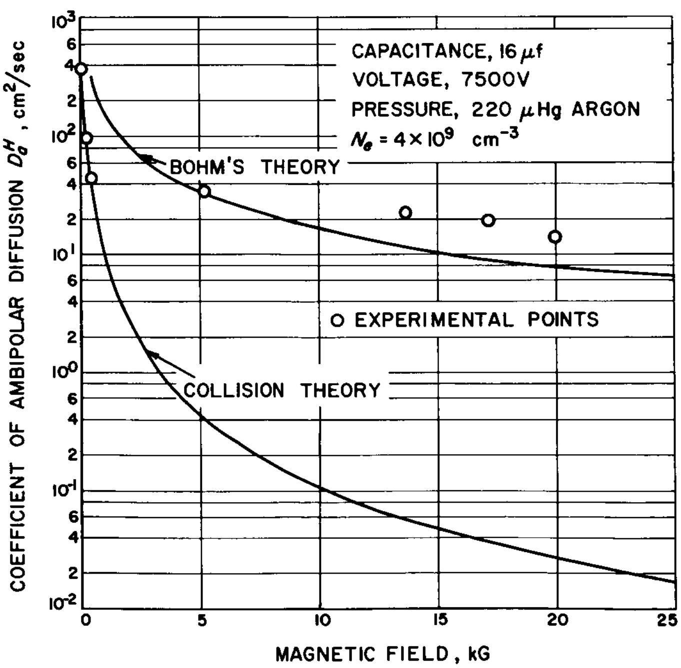


**Fig. 6. Coefficient of ambipolar diffusion as a function of magnetic field at pressure = 220 μ Hg argon**

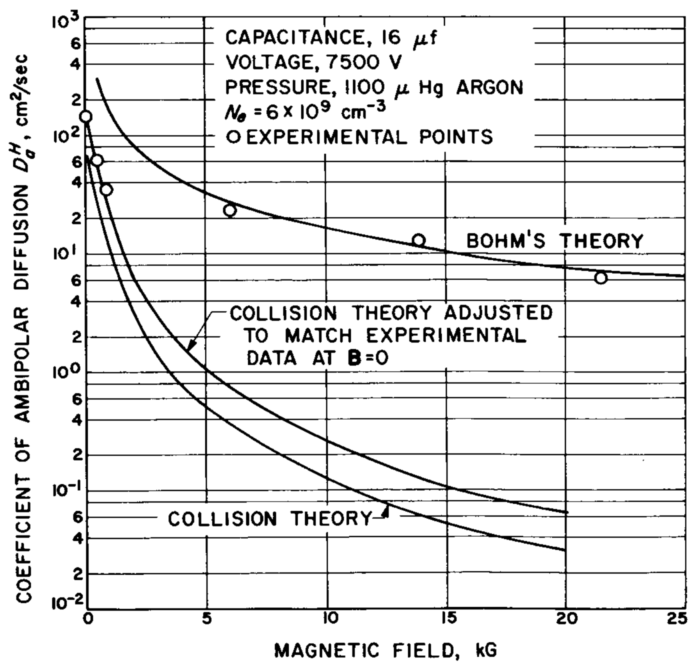


**Fig. 8. Coefficient of ambipolar diffusion as a function of magnetic field at pressure = 1100 μ Hg argon**

The coefficient of diffusion as a function of magnetic field for three different initial pressures, together with theoretical predictions as expressed in Eq. (3) and Bohm's diffusion coefficients (Ref. 2), are shown in Fig. 6, 7, and 8. It is readily seen that the measured $D_a^H$ agrees quite well with the theoretical predictions when the magnetic field is below 1,000 G. As the magnetic field is increased, the measured values are quite different from those predicted by collision theory. In fact, the measured $D_a^H$, in most cases, is more than two orders of magnitude greater than the theory predicted when the magnetic field exceeds 10 kG. In general, the magnetic-field dependency of $D_a^H$ varies as $\mathbf{B}^{-1}$ (Bohm's formulation); in some cases the data show an even flatter distribution (see Fig. 7). When the dependency varies as $\mathbf{B}^{-1}$, the absolute value of $D_a^H$ agrees very well with the theory of Bohm.

## D. Possible Causes of Anomalies

The possible causes for the anomalously rapid loss of the charged particles are investigated as follows:

### 1. Non-Uniformity of Magnetic Field

The radial variation of the magnetic field within the diameter of the discharge tube is undetectable. Axially, the uniformity of the magnetic field is within 5%. The drift of charged particles due to the gradient of the magnetic field under this condition would be negligible in comparison with the decay rate measured.

### 2. Misalignment of the Magnetic Field With the Axis of the Tube

Granichev et al (Ref. 23) have indicated that the effect of misalignment of the magnetic field with the axis of the discharge tube is not equivalent to the shortening of the tube. The effect is associated with the ending of the magnetic lines of force at the wall. Under these conditions it is found that the electrons and ions can diffuse separately and that the diffusion is not ambipolar.

In the present measurement the orientation of the tube is adjusted and the measurement repeated, with no appreciable effect noted.

### 3. Instabilities in the Plasma

The magnetic probe (in this experiment the ratio of plasma pressure to magnetic pressure is in the order of $10^{-2}$) and electrostatic probe both indicate that some disturbances in the plasma exist. The frequencies of these disturbances are measurable. The oscillations are, in general, sinusoidal in shape, or somewhat saw-tooth. The magnetic-field dependency of the frequency is inverse in nature. D'Angelo and Motley (Ref. 24) and Buchelnikova (Ref. 25) have detected similar disturbances but with several frequencies.

It has been shown by several authors (Ref. 10, 26, 27) that a plasma with an inhomogeneous density distribution in a magnetic field can be unstable against a so-called universal instability. The instability can develop in both a low-density and a high-density plasma. The instability leads to an excitation of waves that are essentially perpendicular to the magnetic field but with a finite component along the field. The characteristic frequency can be expressed as

$$\omega = k_y \left( \frac{ckT_e}{eB} \right) \frac{N_e'}{N} \tag{5}$$

where $c$ is the velocity of light, $k_y$ the component of the wave or propagation vector $2\pi/\lambda$ perpendicular to the magnetic field, and $N_e'$ the density gradient. The frequency in Eq. (5) can be estimated by assuming the following constants: wavelength $\lambda \approx$ mean free path $\approx 0.1$ cm, $N_e'/N_e \approx 1/(\text{radius of tube}) \approx 1/1.9$ cm, and $T_e \approx 300°$K. The observed frequencies as a function of $\mathbf{B}$, together with the calculation from Eq. (5), ion-cyclotron frequency, ion oscillation, and ion acoustic wave, are shown in Fig. 9. The frequency of the ion acoustic wave is calculated by assuming that the wavelength is $2.84\pi L_d$ (Debye length) (Ref. 28). Figure 9 indicates, in spite of the crude estimation, that the observed disturbance at high magnetic field agrees with the trend of the so-called universal instability quite well, both in absolute values and $\mathbf{B}$-dependency.

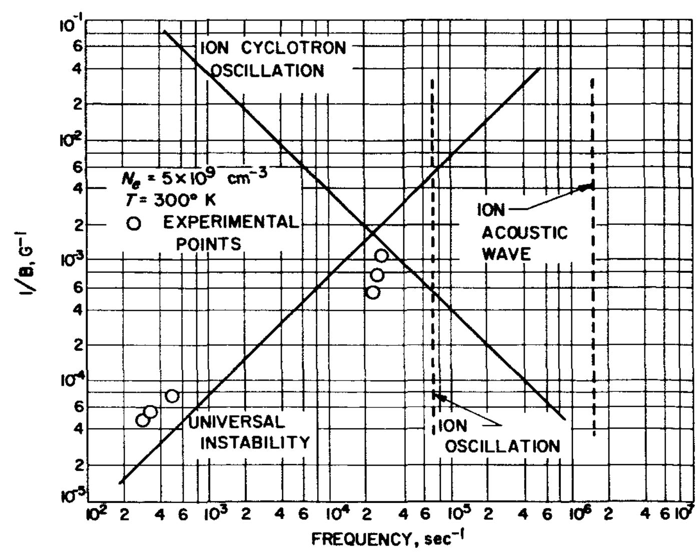


**Fig. 9. Frequencies of instabilities**

# IV. CONCLUSIONS

From the measurements described in this Report, the following conclusions can be deduced:

1. At low magnetic field (lower than 1,000 G) the diffusion of charged particles across a magnetic field agrees with the prediction of collision theory, and in this region of magnetic field, there is no detectable instability in the plasma.
2. At high magnetic field (above 1,000 G) the diffusion is much higher than the collision-theory prediction, and a detectable instability is present in the plasma.
3. The magnitude and **B**-field dependency of the coefficient of diffusion is in good agreement with Bohm's formulation (Ref. 2) in the higher magnetic field region.
4. The general trend of the frequency of the instability and its **B**-field dependency indicates that at high magnetic field ($\approx 10^4$ G) the universal-instability mechanism dominates, whereas in the region of $\approx 10^3$ G the frequency is insensitive to the magnitudes of the magnetic field, indicating the presence of ion oscillations.
5. The possible causes for the anomalous diffusion are thought to be the ion oscillations and universal instability, and the critical field (Ref. 29, 30) for the onset of the instability in this experiment is about 1,000 G.

## ACKNOWLEDGMENT

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